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1993 J. Phys.: Condens. Matter 5 9135

(http://iopscience.iop.org/0953-8984/5/49/016)

J. Phys.: Condens. Matter 5 (1993) 9135–9144. Printed in the UK

Classical mesoscopic conductance fluctuations in systems with boundary scattering

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Received 14 January 1993, in final form 14 September 1993

Abstract. Mesoscopic conductance fluctuations in a 2D electron channel due to correlations between classical boundary scattering processes are discussed. The contribution to conductance provided by the correlations is calculated and analysed; it is a consequence of the inherent roughness of the boundary. As a result stochastic oscillations for a weak magnetic field are exhibited. In the absence of bulk scattering the stochastic behaviour has a fractal structure, the fractal dimension being 1.5. It is shown that at higher temperatures classical mesoscopic fluctuations, discussed here, dominate over the universal conductance fluctuations. Unlike the universal conductance fluctuations, these classical ones are non-universal and reveal a dependence upon the geometry of a sample. Such non-universal behaviour of the classical conductance fluctuations is consistent with recent experimental data observed for ballistic conductors.

1. Introduction

Mesoscopic fluctuations in conductance in systems of small size is a consequence of an inhomogeneous distribution of scatterers. Such fluctuations are due to quantum interference (the so-called universal conductance fluctuations, UCF) [1,2]. However, it has been shown [3,4] that mesoscopic fluctuations can exist within a purely classical picture, i.e. for a classical charged particle moving in a field of random scatterers. Such classical fluctuations contribute to the conductance, which is related to the specific spatial distribution of the scatterers (as in the quantum case). It is obvious that one can describe the sensitivity of the conductance to a specific realization of the scatterers only by taking into account the correlation between successive scattering events¶.

The electron distribution function at the point where a given scatterer is situated is a consequence of the previous acts of scattering, so the probability for an electron to be scattered by this scatterer depends on the spatial arrangement of neighbouring scatterers (note that the effect of this factor on the average conductance was considered by Landauer [5,6]).

The presence of an external magnetic field bends the electron trajectories from straight lines into helices, and changes the correlation in question. This change is of the same type as those due to the variation of the spatial arrangement of the scatterers inside the system.

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¶ Note that in the usual τ approximation leading to the Drude formula such a correlation is averaged out.

0953-8984/93/499135+10$07.50

As a result, a classically weak magnetic field leads to variations in the distribution of carriers in phase space and thus to stochastic-like conductance fluctuations.

The type, amplitude and period of classical mesoscopic fluctuations depend in turn on the parameters of the scattering potential [3, 4]. It appears that, at least for near-ballistic small-size systems with large scatterers (e.g. charged impurities in semiconductors) classical effects can dominate over UCF. On the other hand, one expects classical effects to survive at higher temperatures where quantum effects would be averaged out and unobservable.

The classical mesoscopic fluctuations have been predicted for systems with bulk scatterers [3, 4], but in low-dimensional systems the impurity scattering is usually suppressed [7–9]. In systems with a high mobility of the electron gas, ballistic behaviour of the electrons takes place and the transport properties are very sensitive to the electron scattering at boundary inhomogeneities. Thus, the problem of classical mesoscopic fluctuations induced by boundary scattering is of special importance.

In this paper we consider such 'surface' mesoscopic effects in a narrow electron channel with rough boundaries. A relevant example is realized in a typical laterally restricted electron channel such as fabricated in the GaAs/AlGaAs heterostructure by introducing a confining potential in a two-dimensional electron gas [7,8].

The DC conductivity of a metal film with diffuse boundaries was calculated for the first time by Fuchs [10]. He introduced a phenomenological model of the specularity parameter. The same problem, based on a model of a statistically rough surface, was considered in [11]. This model assumes that inhomogeneities are distributed randomly and continuously along the surface (see, e.g., [12]). Statistical properties of roughnesses are characterized by two parameters: the root-mean-square height and the mean length of the roughnesses. The quantum mechanical approach to the resistivity of a thin film with a rough surface was developed in [13–15].

In quantum wires the boundary scattering is usually specular, but a small probability of non-specular reflection gives rise to some anomalous phenomena. In particular, in quasi-one-dimensional wires, boundary scattering leads to weak localization of electronic states [16–19] and to anomalies in low-field magnetoresistance [20–23]. Thus boundary scattering is essential for the transport properties of a narrow electron waveguide. The origin of boundary roughness is related to variations in the width of a channel and follows the fluctuations of the confinement potential [24]. Usually the amplitude of the boundary roughness is small compared to the width of a channel. This allows one to consider a channel with a constant width and the boundary inhomogeneities manifest themselves through random surface scattering of electrons.

As shown in [25–27] surprisingly many effects in narrow electronic waveguides can be understood with a purely classical billiard ball model [28] if the number of transverse channels exceeds three. In what follows we will discuss the conditions for purely classical description of mesoscopic fluctuations. We note that classical aspects of boundary scattering are physically appropriate if the typical scale of the boundary roughnesses exceeds the de Broglie wavelength.

The classical billiard ball model [28] deals with an electronic trajectory which for a given channel is a broken line. It is obvious that, for a random boundary, such a trajectory has a random character. Each trajectory gives a certain contribution to the conductance of a channel. This contribution depends upon the coordinates of the last collision with the boundary. By averaging over these coordinates the conductance is averaged over a random distribution of roughnesses. This averaged conductance is the value which has been previously calculated [10, 11, 13, 14].

This conductance depends upon the statistical parameters of the random boundary (for example, in Fuchs' model [10] it is the specularity parameter and in the model of a rough boundary [11] these are the RMS height and the length of the roughnesses). Macroscopically identical samples have the same averaged conductance. But macroscopically identical samples possess different microscopical arrangements of scatterers and thus different sets of trajectories. This difference of the microscopic properties should manifest itself in the small irregular corrections to the averaged conductance. In the macroscopic limit these corrections vanish, but for the small-size samples they are essential to the classical mesoscopic fluctuations of the conductance [3,4].

In order to observe these fluctuations it is not necessary to consider an ensemble of macroscopically identical samples. It is experimentally more convenient to deal with the same sample (i.e. with the same arrangement of scatterers) but change the electron trajectories by applying a weak external magnetic field.

In a weak external magnetic field the points at which an electron collides with the rough boundary are shifted, i.e. the distribution of scatterers along the electron trajectory is changed. So for a given value of magnetic field one has a special arrangement of scatterers in the sample. When the magnetic field varies continuously, small-amplitude stochastic oscillations of the conductance arise. The pattern of these oscillations is determined by the specific arrangement of the irregularities along the boundary of the channel, and is thus an individual characteristic of the sample (a 'magnetofingerprint').

The purpose of this paper is to estimate a typical amplitude and period of classical mesoscopic fluctuations induced by scattering of the conduction electrons at the rough boundary of a narrow electron channel. In section 2 we solve the kinetic equation and obtain the formula for the conductance which is a series representation of the number of electron collisions with rough boundaries. In section 3 we discuss the influence of a weak transverse magnetic field and calculate the amplitude and period of the oscillations. Here we also demonstrate that stochastic oscillations of conductance have a fractal structure if bulk scatterers are absent. The case of almost specular reflection of electrons from the rough boundaries is discussed in section 4. Comparison of classical fluctuations with universal quantum fluctuations as well as recent experiments with our results are given in section 5.

2. Mesoscopic corrections to conductance

Consider a 2D electron channel of mean width a (figure 1). The sizes of the boundary irregularities (RMS height and mean length) are assumed to be small compared to a. Boundary roughness leads to random scattering of conduction electrons. We now consider how a current is affected by this roughness.

The current density, j_x, in this channel can be written in the following form:

$$j_x(x,y) = 2e \int \frac{\mathrm{d}^2 \boldsymbol{p}}{(2\pi\hbar)^2} v_x \frac{\partial f_0}{\partial \varepsilon} \chi. \tag{1}$$

Here e is the charge of an electron, $\chi(\partial f_0/\partial\varepsilon)$ is the non-equilibrium addition to the Fermi distribution function, $f_0(\varepsilon)$, and y is the inner normal to the averaged boundaries of the sample (figure 1).

The non-equilibrium distribution function χ is the solution of the Boltzmann equation taking account of the bulk scattering in the τ approximation:

$$\frac{\chi}{\tau} + v_x \frac{\partial \chi}{\partial x} + v_y \frac{\partial \chi}{\partial y} = e\boldsymbol{E} \cdot \boldsymbol{v} = eEv_x. \tag{2}$$

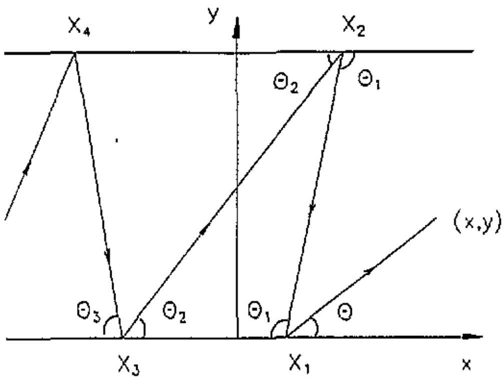

Figure 1. Trajectory of an electron in a 2D electron channel with diffuse boundaries.

This equation can be solved by the method of characteristics. Let the angles of the electron impact with the surface be $\theta, \theta_1, \theta_2, \cdots, \theta_n$, and the coordinates of the impact points be $X_1, X_2, \cdots, X_n$ (figure 1). Between the points with coordinates X_n and X_{n+1} an electron moves with constant velocity $v_x = v_F \cos\theta_n$, $v_y = v_F \sin\theta_n$, where v_F is the Fermi velocity. Making use of this notation one can write the solution of (2) as

$$\chi = e\int_{X_1}^{x} dx' E \exp\left(\frac{x'-x}{l\cos\theta}\right) + e\sum_{n=1}^{\infty}\int_{X_{n+1}}^{X_n} dx' E \exp\left[-\frac{1}{l}\left(\frac{x-X_1}{\cos\theta} + \frac{X_1-X_2}{\cos\theta_1} + \cdots + \frac{X_n-x'}{\cos\theta_n}\right)\right]. \tag{3}$$

Here $l = v_F\tau$ is the mean free path.

For the case where there is a strong size effect, i.e.

$$l >> a \tag{4}$$

then the mesoscopic fluctuations manifest themselves most profoundly. It is obvious that the main contribution to the sum over n in (3) is due to terms with $n \leqslant N$, where

$$N = l/a >> 1. \tag{5}$$

In these terms we can neglect the difference of the exponent from unity and then rewrite the sum in the following way:

$$\int_{X_1}^{x} dx' + \int_{X_2}^{X_1} dx' + \cdots + \int_{X_{n+1}}^{X_n} dx' = y\cot\theta + a(\cot\theta_1 + \cot\theta_2 + \cdots + \cot\theta_n). \tag{6}$$

The local reflection law at the point $x = X_n$ of the rough surface gives the relation between the angles θ_{n-1} and θ_n. In the general case

$$\theta_{2n+1} = \Psi_b(\theta_{2n}, X_{2n+1}) \qquad \theta_{2n} = \Psi_u(\theta_{2n-1}, X_{2n}) \tag{7}$$

where the functions Ψ_b and Ψ_u determine the reflection laws from the bottom and upper channel boundaries, respectively. The dependence of Ψ_b and Ψ_u on impact angle is considered to be regular (e.g. in accordance with law of optics) but not spatially regular. The characteristic scale of the change of Ψ_b and Ψ_u coincides with a mean length of roughness

L. Thus (3), (6) and (7) completely determine the non-equilibrium distribution function, χ. Making use of (1), (3) and (6) we can now obtain the current density at the point (x, y):

$$j_x(x,y) = 2e^2 a \int \frac{\mathrm{d}^2 p}{(2\pi\hbar)^2} v_x E \frac{\partial f_0}{\partial \varepsilon}\left(\frac{y}{a}\cot\theta + \cot\Psi_\mathrm{b}(\theta, X_1) + \cot\left[\Psi_\mathrm{u}\left(\Psi_\mathrm{b}(\theta, X_1), X_2\right)\right] + \cdots\right) + (\Psi_\mathrm{b} \rightleftharpoons \Psi_\mathrm{u}). \tag{8}$$

The first term in the large round brackets describes the averaged conductivity of a sample with a diffuse boundary; it accounts for the section of electron trajectory $y\cot\theta$ after the last collision with the rough boundary. Integration of this term over θ gives the singular logarithmic factor $\ln(v_\mathrm{F}\tau/a)$, in agreement with Fuchs' result [10]. The second term describes the fluctuations in the electron flow connected with electron scattering at the point $x = X_1$. It depends only on the coordinate X_1, and obviously it does not contain any information about the correlation between successive scattering events of the electron at the boundary. Such information is contained in the third term which depends on two coordinates, X_1 and X_2. In what follows we will show that it is this term that gives the main contribution to mesoscopic fluctuations in an external magnetic field. The correlation terms of higher order, depending on three and more coordinates, also gives a contribution to the stochastic oscillations, but the amplitude of these oscillations turns out to be rather small. In calculating the integral in (8) the coordinates X_1 and X_2 should be considered as functions of x and y:

$$X_1 = x - y\cot\theta \qquad X_2 = x - y\cot\theta - a\cot[\Psi_\mathrm{b}(\theta, x - y\cot\theta)]. \tag{9}$$

Substituting (9) into (8) one can see that the third (correlation) term is a random function of the composite argument, which in turn contains a random function. The correlation scale of this composite function, with respect to the variable x, is L^2/a, which is many times less than L because $a/L \gg 1$. This property is a consequence of the general principle of the initial condition memory loss due to random scattering. The higher-order terms omitted in (8) correspond to trajectories with multiple surface scattering and have a correlation scale $(a/L)^n$ times less than L (n is the number of collisions). Due to integration over momentum in (8) these terms give a relatively small contribution to the mesoscopic fluctuations of conductance because of rapid oscillations of random functions in the integrand. However, these terms are important for the fractal structure of mesoscopic oscillations, as shown below.

To estimate the integrals in (8) we take account of the fact that the integration of a random function leads to a random function, but to one with a larger correlation scale. Consider the random function $\varphi(x)$ with correlation scale x_0, with an amplitude of the order of unity and with a zero mean value. It is obvious that

$$\int_\alpha^{\delta+\alpha} \varphi(x)\,\mathrm{d}x = \sqrt{\delta x_0}\,\gamma(\alpha,\delta) \qquad \delta \gg x_0 \tag{10}$$

where $\gamma(\alpha, \delta)$ is a random function with an amplitude of the order of unity and a characteristic scale over α and δ of the order of δ.

Using (10) we perform the integration over θ in (8) and obtain the current density:

$$j_x(x,y) = \sigma_0 E \frac{a}{v_\mathrm{F}\tau}\left(\frac{y}{a}\ln\frac{v_\mathrm{F}\tau}{a} + \sqrt{\frac{L}{a}}\gamma_1(x,y) + \frac{L}{a}\gamma_2(x,y) + \cdots\right). \tag{11}$$

Here $\sigma_0 = p_F^2 e^2 m / 2\pi\hbar^2\tau$ is the 2D conductivity of the electron gas, γ_1 and γ_2 are random functions with a characteristic scale of oscillations over x and y of the order of a, p_F is the Fermi momentum and m is the effective electron mass. The length of the roughnesses, L, is assumed to be much less than the channel width, a; therefore (11) is the asymptotic series for the current density with respect to the small parameter $(L/a)^{1/2}$.

The current density in (11) has a small contribution depending on the coordinates x and y. In general, this contribution does not satisfy the electrical neutrality condition $\mathrm{div}\boldsymbol{j} = 0$. This condition holds due to a presence of some fluctuating electric field $\boldsymbol{E}'$. This field can be determined directly from the neutrality condition by substituting into (1) and (2) the total field $\boldsymbol{E} + \boldsymbol{E}'$ instead of the averaged field. The spatial scale of the fluctuations of the current (11) is given by the statistical properties of random functions γ_1, γ_2, and thus is of the order of a. However, it follows from the condition $\mathrm{div}\boldsymbol{j} = 0$ that the field $\boldsymbol{E}'$ should have the same spatial scale of fluctuations. Small-scale fluctuations of the electric field arise only in the narrow 'rough' layer near the channel boundaries. The width of this layer is of the order of the mean height of the roughness, ξ. Since we are considering a channel of constant width a (i.e. $\xi \ll a$) we can neglect the contribution of this 'rough' layer to the conductance of the channel. Thus the electrical neutrality condition does not affect the statistical characteristics of conductance oscillations and we can readily neglect it.

The average current density can be obtained from (11) after integration over the volume of the sample. Assuming that the length of the channel b is much larger then the width a ($b \gg a$) and using (10) we obtain the following fluctuating corrections to the conductance of a channel:

$$\frac{\Delta G}{G} = \left(\frac{a}{b}\right)^{1/2}\left(C_1\sqrt{\frac{L}{a}} + C_2\frac{L}{a} + \cdots\right)[\ln(l/a)]^{-1}. \tag{12}$$

Here $C_1, C_2, \ldots$ are coefficients of the order of unity, depending on the realization of the rough boundary.

$$G = \frac{e^2}{\hbar}\frac{a^2}{b\lambda}\ln(l/a)$$

is the conductance of the 2D electron channel with diffuse boundaries, and $\lambda = 2\pi\hbar/p_F$ is the de Broglie wavelength of the electron.

Note that though the first term in (12) dominates it can be observed experimentally only with a variation of the sample size (for example, variation of its width a or the shape of the whole rough boundary). For the 2D electron gas such a variation can be realized by a variation of the gate voltage. Local variations of the boundary of a distance of the order of the roughness of length L lead to the following variation of conductance:

$$\frac{\Delta G}{G} \sim \left(\frac{a}{b}\right)^{1/2}\frac{L}{a} \tag{13}$$

which corresponds to the second term in (11).

3. Mesoscopic oscillations of conductance in a weak magnetic field

Let us consider now the influence of a weak magnetic field, $\boldsymbol{H}$, applied perpendicular to the plane x, y. The essential influence of a magnetic field on the averaged conductivity (the first term in (8)) appears if

$$(Ra)^{1/2} < l \tag{14}$$

where R is the Larmor radius. Condition (14) means that the magnetic field is rather strong: a typical electron trajectory does not touch one of the channel's boundaries. However, a much weaker magnetic field can influence the fluctuating terms in (8). In a weak magnetic field we can consider the electron trajectories to be straight lines, as before. But due to the magnetic field, weak bending small corrections are given as

$$\Delta X_1 = \frac{y^2}{R\sin^3\theta} \sim \frac{a^2}{R} \qquad \Delta X_2 \sim \frac{a}{L}\Delta X_1, \cdots, \Delta X_n \sim \left(\frac{a}{L}\right)^n \Delta X_1 \tag{15}$$

which appear for the coordinates $X_1, X_2, \ldots, X_n$. The correction ΔX_1 in (8) appears only in combination with x, and one can consider it as a displacement of the observation point. After averaging over the volume of a sample the effect associated with the variation of X_1 vanishes. This means that the first correction in (8) does not depend on a weak magnetic field. On the other hand, the presence of the correction ΔX_2 changes the correlation between successive impact points so if $\Delta X_2 \geqslant L$ one has the same effect as in the case of the complete variation of the rough surface. One can conclude that in a magnetic field obtained from the condition

$$\Delta X_2 \geqslant L \tag{16}$$

the conductance fluctuates with the amplitude given by (13). Condition (16) holds in a magnetic field $H \geqslant H_{\mathrm{cl}}$, where H_{cl} is the magnetic field strength corresponding to the displacement $\Delta X_2 = L$. Making use of (15) we get the characteristic period of oscillations:

$$H_{\mathrm{cl}} = \frac{cp_{\mathrm{F}}}{e}\frac{L^2}{a^3} = \frac{\Phi_0}{\lambda}\frac{L^2}{a^3} \tag{17}$$

where $\Phi_0 = 2\pi\hbar c/e$ is the fundamental magnetic flux. Note that (17) does not contain Planck's constant and is a solely a classical effect.

Equation (17) gives the fundamental period of stochastic oscillations of conductance in a weak magnetic field. The background of these oscillations contain a fine structure with a period a/L times less than those of (17), whose amplitude is $(a/L)^{1/2}$ times less than (13). The fine structure is described by the first omitted term in (8). It depends on three coordinates: X_1, X_2 and X_3. The next term depending on X_1, X_2, X_3 and X_4 gives the superfine structure with period and amplitude decreasing according to the same law, and so on. If we take account of the infinite number of terms, i.e. the infinite number of electron collisions with the boundaries, then we come to conclusion that $\Delta G(H)$ has a self-similar structure. Such a curve is a stochastic fractal (see, e.g., [29]). It is easy to obtain that the fractal dimension of the curve $\Delta G(H)$ is 1.5.

However, we would like to note that so far as the number of collisions (5) is limited by the finite bulk free path the smallest period and amplitude of the oscillations are of the order of $H_{\mathrm{cl}}(L/a)^N$ and $(a/b)^{1/2}(L/a)^{N/2}$. So for the real samples at the smaller scale the curve $\Delta G(H)$ has no scaling behaviour.

Stochastic oscillations of the conductance, with amplitudes given by (13) and periods by (17), are classical manifestation of microscopic properties of a mesoscopic sample and they can be considered as a classical magnetofingerprint. They appear when the magnetic field H exceeds H_{cl}. It is interesting to compare the value of the period of the classical oscillations, H_{cl}, with the value of the magnetic field

$$H_l = \frac{cp_{\mathrm{F}}}{e}\frac{a}{l^2} \tag{18}$$

calculated from (14). The ratio

$$\frac{H_{\mathrm{cl}}}{H_l} = \left(\frac{L}{a}\right)^2 \left(\frac{l}{a}\right)^2 \tag{19}$$

is a product of a small parameter $(L/a)^2$ and a big parameter $(l/a)^2$. This ratio is small if

$$(L/a)^2 \ll (a/l)^2. \tag{20}$$

The last inequality is true, since the parameter L is microscopic while l and a are macroscopic quantities. However, even if it does not hold then classical mesoscopic oscillations with the same amplitude and period should appear in the background of the smooth dependence of $G(H)$.

4. Almost specular reflection

We now consider the mesoscopic fluctuations in the channel with almost specular boundaries. In such a case the functions Ψ_{u} and Ψ_{b} have a large deterministic (specular) part and a small random contribution:

$$\Psi(\theta, X) = \theta + \alpha(\theta, X) \qquad \alpha \ll \theta. \tag{21}$$

A typical value of the random function α coincides with the angular width of the scattering indicatrix. The scattering indicatrix as a function of height and length of roughness has been obtained in [12, 30]. In the almost-specular approximation one can expand the cotangents in (8) in terms of the small random function α. After this the sum of all the deterministic terms should be changed by the free path l. This sum forms the conductivity of the channel to a zero approximation and for specular reflection the conductivity coincides with σ_0. The correction terms arising after expansion of the cotangents should be averaged with the help of (10). However, we should consider the fact that the typical amplitude of the random function (i.e. α) is not unity. As a result, we get (12) for the fluctuating part of the conductance (to within a logarithmic factor). Now the values of constants $C_1, C_2, \ldots$, which coincide with the angular width of scattering indicatrix are not of order unity. The angular width of the scattering indicatrix for the near specular reflection is much less than that for the diffuse reflection. This causes the amplitude of stochastic oscillations (which is proportional to $C_1, C_2, \ldots$) to decrease in the channels with almost specular boundaries.

The classical motion of the electron along the trajectory considered in this paper corresponds to the Kirchhoff approximation in the theory of wave scattering from a rough surface [12]. In this approximation the electron wavepacket is reflected almost specularly from the random surface if $\Gamma \ll 1$ and diffusely if $\Gamma \gtrsim 1$ [30]. Here $\Gamma = \xi/L$ is the mean slope of the roughness. In the Kirchhoff approximation the angular width of the scattering indicatrix coincides with Γ [30]. Thus the amplitude of the stochastic mesoscopic fluctuations of conductance decreases in proportion to the parameter ξ/L. For a sample with a perfect boundary ($\xi/L = 0$) oscillations vanish, and for a diffuse boundary ($\xi/L \gtrsim 1$) oscillations have the maximum amplitude given by (13).

5. Classical and quantum fluctuations of conductance

Before comparing the present results with those expected from quantum effects we note that a deterministic classical trajectory only holds if the diffraction effects can be neglected. Estimates of the diffraction divergence of the electronic beam scattered by a roughness element provides the condition for the correlation between successive scattering events to be of a purely classical nature:

$$L > \sqrt{\lambda a} > \lambda. \tag{22}$$

Note that the presence of boundary roughness with a scale larger than the de Broglie wavelength seems to be typical for 2D channels [31]. Accordingly, we can write the final chain of inequalities which are necessary for the classical consideration to be applicable in the ballistic regime:

$$\lambda < \sqrt{\lambda a} < L \ll a \ll l. \tag{23}$$

Universal quantum fluctuations of conductance [1,2] in disordered samples are size independent and have an amplitude of order e^2/h. Using (13) one obtains the ratio of the amplitudes of the conductance fluctuations in the quantum limit to that in the classical limit as

$$\frac{\Delta G}{(e^2/h)} = \frac{L}{\lambda}\left(\frac{a}{b}\right)^{3/2}. \tag{24}$$

This formula shows that in a realistic situation ($b/a \sim 10$) classical mesoscopic effects can be at least as large as quantum effects and even will dominate if $L/\lambda \gtrsim 30$. On the other hand, classical effects can be present at higher temperatures where inelastic scattering destroys phase coherency and quantum effects are of no importance.

Universal quantum fluctuations have been observed in the magnetoresistance of disordered metals (for a review see, e.g., [32]) for the diffuse regime of electron motion in the bulk. At low temperatures they are independent of both the degree of disorder and the sample geometry. However, very recently size-dependent oscillations have been observed in GaAs/AlGaAs heterostructures [33]. The regime of electron motion was ballistic rather than diffuse ($l \gtrsim a$). The amplitude of the conductance fluctuations in a weak magnetic field exhibited the essential dependence on the gate voltage, i.e. on the width of the sample. For rather narrow channels corresponding to the ballistic regime ($a = 0.05$–$0.25\,\mu$m) this dependence was very near to $a^{3/2}$. The same dependence is given by our formula (24). Note that the quantum calculations make no such prediction.

We would like to note that quantum conductance fluctuations in the ballistic regime in quantum wires were calculated numerically [34]. It was shown that size-quantization leads to a non-universal character of the fluctuations: they exhibit a dependence upon the length of the channel. Unfortunately these numerical results [34] are not enough to provide a comparison of the amplitude of quantum and classical fluctuations as well as to determine the dependence of the amplitude of quantum fluctuations on the channel width.

In summary, the present theory provides a description of classical conductance fluctuations induced by the random scattering of conduction electrons at the boundaries of a 2D electron channel. Unlike the universal quantum fluctuations the classical fluctuations are size dependent and persist to rather high temperatures.

Acknowledgments

AK wishes to thank Consejo Nacional de Ciencia y Tecnología for a Cátedra Patrimonial grant during 1992/93.

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